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Unit- 7. Alternating Current

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If you want to shine like a Sun, first burn like the Sun.

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7. Alternating Current

Most of the electric power generated and used in the world is in the form of alternating current (a.c.). This is because-

(i) Alternating voltage can easily and efficiently be converted from one value to other by means of transformers.

(ii) The Alternating current energy can be transmitted and distributed over long distances economically without much loss of energy.

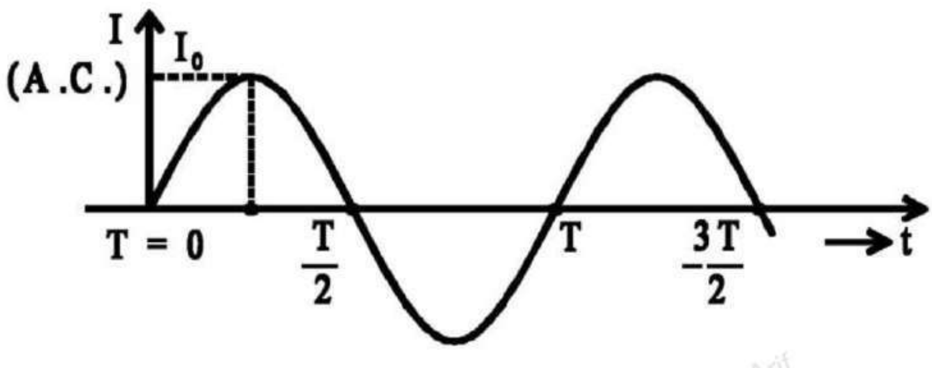
Alternating Current (a.c.)

The magnitude of alternating current (a.c.) changes continuously with time and its direction is reversed periodically.

It is represented by -

$\boxed{I = I_0 \sin\omega t}$ or $\boxed{I = I_0 \cos\omega t}$

Here I is instantaneous value of current and I_0 is the peak value or maximum value of a.c. ω is called angular frequency of a.c.



Where $\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$ $\frac{1}{T} = f$ (frequency)

Where T is the time period of a.c. It is equal to the time taken by the a.c. to go through one complete cycle of variations.

Also f is the frequency of a.c. It is equal to the number of complete cycles of variations gone through by the a.c. in one second.

Similarly the alternating e.m.f. may be represented by-

$$\boxed{E = E_0 \sin\omega t} \quad \text{or} \quad \boxed{E = E_0 \cos\omega t}$$

In a.c. circuits, two additional circuit elements are used. These are inductor (L) and capacitor (C). Thus current and voltage in a.c. circuit are controlled by three circuit elements R, L and C.

* The voltage across a pure inductor L is given by-

$$\boxed{V = L\frac{dI}{dt}}$$

where dI/dt is rate of change of current.
The above equation shows that an inductor affects the voltage only when current I changes with time.

* For an ideal capacitor of capacitance C,

$$Q = CV$$

As $I = \frac{dQ}{dt} \Rightarrow I = \frac{d(CV)}{dt}$

or

$$\boxed{I = C\frac{dV}{dt}}$$

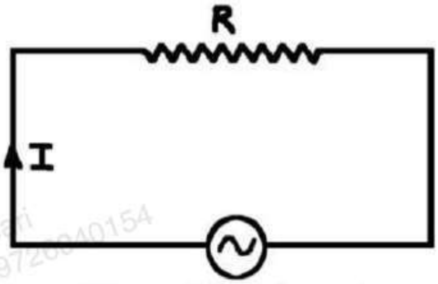
where dV/dt is rate of change of potential.
The equation shows that a capacitor affects the current only when V changes with time

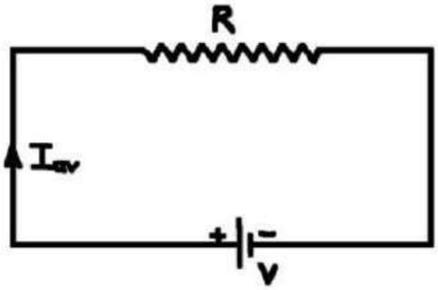
Nikola Tesla   Thomas Edison

Alternating Current	Direct Current
AC is easy to be transferred over longer distances – even between two cities – without much energy loss.	DC cannot be transferred over a very long distance. It loses electric power.
The rotating magnets cause the change in direction of electric flow.	The steady magnetism makes DC flow in a single direction.
The frequency of AC is dependent upon the country. But, generally, the frequency is 50 Hz or 60 Hz.	DC has no frequency or zero frequency.
In AC the flow of current changes its direction forward and backward periodically.	It flows in a single direction steadily.
Electrons in AC keep changing their directions – backward and forward.	Electrons only move in one direction – forward.

Mean or Average value of Alternating Current

It is defined as that value of steady current which would send the same amount of charge through a circuit in the time of half cycle (i.e. $T/2$) as is sent by the a.c. through the same circuit, in the same time.





$I = I_0 \sin\omega t$

> * The mean or average value of a.c. over one complete cycle is zero. So we can calculate the mean or average value of a.c. over any half cycle.

Let an alternating current be represented by-

$$\boxed{I = I_0 \sin\omega t}$$

If the strength of current is assumed to remain constant for a small time dt, then small amount of charge sent in a small time dt is -

$$dq = I\,dt$$

Let q be the total charge sent by a.c. in the first half cycle $(0 \rightarrow T/2)$

$$q = \int_0^{T/2} I\,dt \quad \text{or} \quad q = \int_0^{T/2} I_0 \sin\omega t\,dt$$

$$q = I_0 \left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2} \qquad [\omega T = 2\pi]$$

$$= -\frac{I_0}{\omega}\left[\cos\omega\frac{T}{2} - \cos 0^\circ\right]$$

$$= -\frac{I_0}{\omega}\left[\cos\pi - \cos 0^\circ\right]$$

$$\boxed{q = \frac{2 I_0}{\omega}} \quad \text{— (1)}$$

If I_{av} represents the mean or average value of a.c. over the 1st half cycle then -

$$\boxed{q = I_{av} \times \frac{T}{2}} \quad \text{— (2)}$$

from equation (1) and (2) we get -

$$I_{av} \times \frac{T}{2} = 2\frac{I_0}{\omega} = \frac{2I_0}{2\pi/T}$$

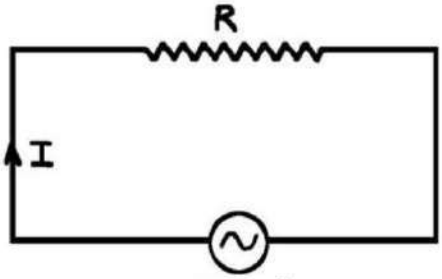
$$\boxed{I_{av} = \frac{2I_0}{\pi} = 0.637\, I_0}$$

Hence mean or average value of a.c. over positive half cycle is 0.637 times the peak value of a.c., or we can say 63.7 % of the peak value.

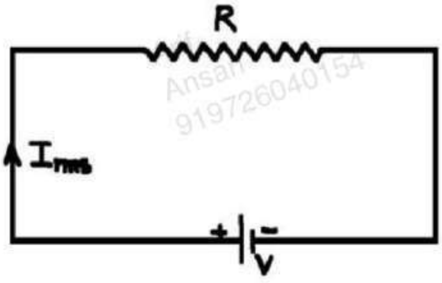
Note - The ordinary d.c. ammeter and d.c. voltmeter cannot measure alternating current/voltage. They record zero reading, when used in a.c. circuit, because average value of alternating current/voltage over a full cycle is zero.

Root Mean Square Value of Alternating Current

It is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the a.c., when passed through the same resistance for the same time.



$I = I_0 \sin\omega t$



The r.m.s value is also called effective value of a.c. or virtual value of a.c. It is represented by I_{rms} or I_v.

Let an alternating current is represented by -

$$\boxed{I = I_0 \sin\omega t} \quad — (1)$$

Let this current flow through a resistance R. In a small time dt, the amount of heat produced in resistance R is -

$$dH = I^2 R\, dt \quad — (2)$$

In one complete cycle (time $0 \to T$), the total amount of heat produced in the resistance R is -

$$H = \int_0^T I^2 R\, dt$$

or

$$H = \int_0^T (I_0^2 \sin^2\omega t) R\, dt$$

$$H = I_0^2 R \int_0^T \frac{(1 - \cos 2\omega t)}{2}\, dt$$

$$H = \frac{I_0^2 R}{2}\left[\int_0^T 1 \cdot dt - \int_0^T \cos 2\omega t\, dt\right]$$

$$H = \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$H = \frac{I_0^2 R}{2}\left[T - \frac{\sin 2\omega T}{2\omega}\right] \qquad (\because \omega T = 2\pi)$$

So

$$H = \frac{I_0^2 R T}{2} \quad — (3)$$

If r.m.s. value or virtual value of a.c. is represented by I_{rms}, then the amount of heat produced in the same resistance R in the same time T would be -

$$H = I_{rms}^2 R T \quad — (4)$$

So from eq. (3) and (4) -

$$I_{rms}^2 RT = \frac{I_0^2 RT}{2}$$

$$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0}$$

Hence the r.m.s. value or virtual value or effective value of a.c. is 0.707 times the peak value of a.c. or we can say 70.7 % of the peak value of a.c.

> **Note-** Alternating current / voltage are measured by a.c. ammeter / voltmeter respectively.
> These instruments are called hot wire instruments and they measure only virtual or r.m.s. values of alternating currents / voltages.

Q. (CBSE 2012) A light bulb is rated 150 W for 220 V AC supply of 60 Hz. Calculate (a) the resistance of the bulb (b) the rms current through the bulb.

Sol. Given that - $P = 150\,W$, $V = 220\,V$

(a) Resistance of the bulb $R = \frac{V^2}{P}$

$$R = \frac{220 \times 220}{150} = 322.7\ \Omega$$

(b) $$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{322.7} = 0.68\ A$$

A.C. Circuit Containing Resistance Only

Let a source of alternating e.m.f. is connected to a pure resistance R.

Suppose the alternating e.m.f. supplied is represented by -

$$\boxed{E = E_0 \sin\omega t} \quad \text{— (1)}$$

Let I be the current in the circuit at any instant t. The potential difference developed across R will be IR.

This must be equal to e.m.f. applied at that instant -

$$IR = E = E_0 \sin\omega t$$

or

$$\boxed{I = \frac{E_0}{R}\sin\omega t = I_0 \sin\omega t} \quad \text{— (2)}$$

where $I_0 = E_0/R$ (Maximum value of current)

★ The behaviour of R in d.c. and a.c. circuits is the same. R can reduce a.c. as well as d.c. equally effectively.

By comparing equation (1) and (2) we can see that in an a.c. circuit containing R only, the voltage and current are in the same phase.

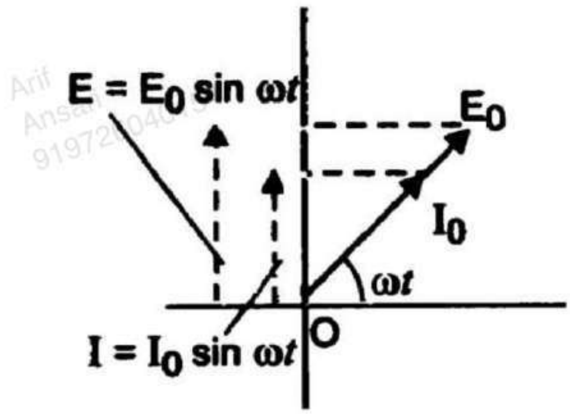
Phasor Diagram-

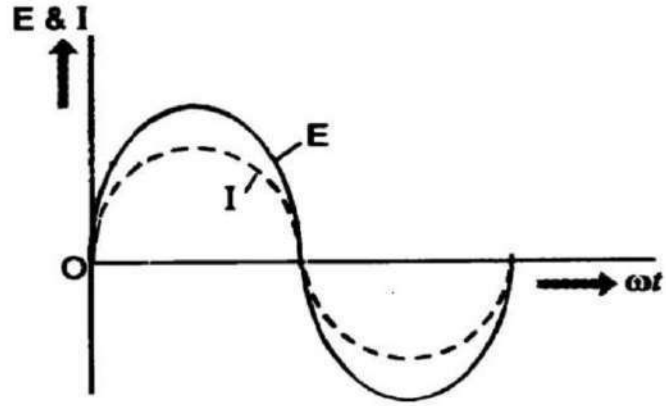
A rotating vector that represents a quantity varying sinusoidally with time is called a phasor.

(i) In phasor diagram, peak values of alternating current (I_0) and alternating e.m.f. (E_0) are represented by arrows called phasors.

(ii) They are inclined to horizontal axis at $\angle \omega t$ and rotate in the anticlockwise direction.

The length of the arrow represents the maximum value of the quantity i.e. I_0 and E_0.





(iii) The projection of the arrow on any axis represents the instantaneous value of the quantity.

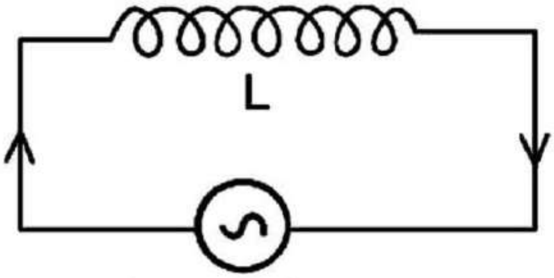
(iv) The phasor difference between the two alternating quantities is represented by the angle between the two vectors $\vec{E_0}$ and $\vec{I_0}$.

(v) In the a.c. circuit containing R only, current and voltage are in the same phase i.e. the phase angle between alternating voltage and current through R is zero.

A.C. Circuit Containing Inductance only

Let a source of e.m.f. is connected to a circuit containing a pure inductance L only. Suppose the alternating e.m.f. supplied is given by-

$$\boxed{E = E_0 \sin\omega t}$$



$E = E_0 \sin\omega t$

If dI/dt is the rate of change of current through L at any instant, then induced e.m.f. in the inductor at the same instant is-

$$\boxed{e = -L\frac{dI}{dt}}$$

The negative sign indicates that induced e.m.f. oppose the change of current.

To maintain the flow of current in this circuit, applied voltage must be equal and opposite to the induced voltage.

$$E = -\left(-L\frac{dI}{dt}\right) = E_0 \sin\omega t$$

or $$dI = \frac{E_0}{L}\sin\omega t \, dt$$

By integrating both sides we get-

$$\int dI = \frac{E_0}{L}\int \sin\omega t \, dt$$

$$I = \frac{E_0}{L}\left(\frac{-\cos\omega t}{\omega}\right) = \frac{-E_0}{\omega L}\cos\omega t$$

$$I = -\frac{E_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right)$$

or $$I = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

The current (I) will be maximum (I_0), when

$$\sin(\omega t - \pi/2) = \text{maximum} = 1$$

So $$\boxed{I_0 = \frac{E_0}{\omega L}}$$

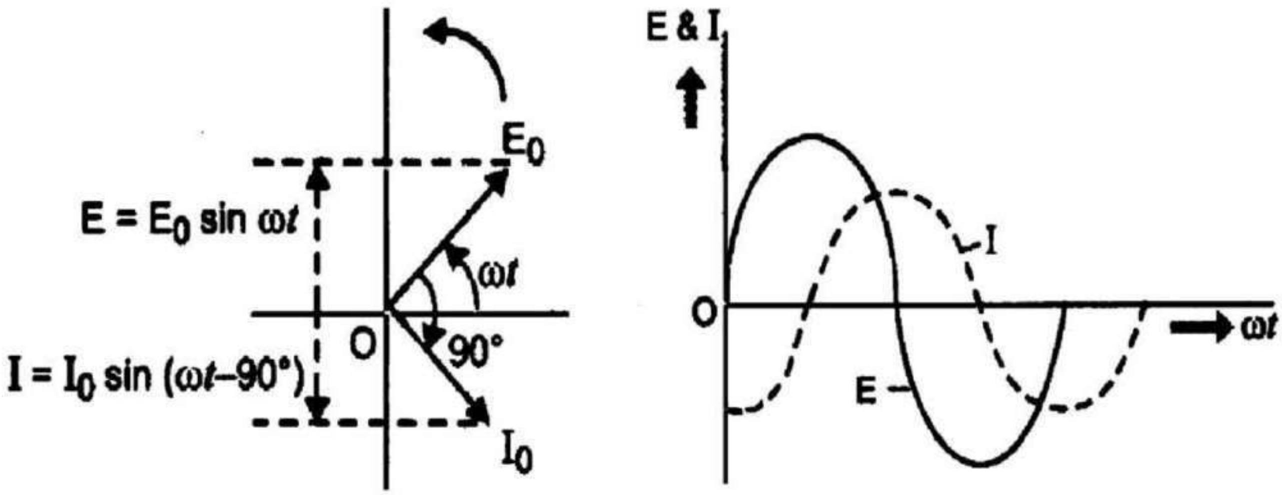
Hence $$\boxed{I = I_0 \sin(\omega t - \pi/2)}$$

So we find that in an a.c. circuit containing L only, alternating current I Lags behind the alternating voltage E by a phase angle of 90°. or we can say one fourth of a period.

Phasor Diagram -

In the phasor diagram of a.c. circuit containing L current Lags behind the e.m.f. by 90°, so the phasor representing I_0 is turned clockwise through 90° from the direction of E_0.

The projections of these phasors on y axis give instantaneous values E and I.



Here ωL represents the effective resistance offered by inductance L. This is called inductance reactance and is denoted by X_L.

So $$\boxed{X_L = \omega L = 2\pi f L}$$ $f \rightarrow$ frequency of a.c. supply

* In dc circuits $f = 0$ so $X_L = 0$

so that a pure inductance offers zero resistance to d.c. It means a pure inductor cannot reduce d.c.

SI unit of inductive reactance is ohm. (Ω)

$$X_L = \omega L = \frac{\text{henry}}{\text{sec.}} = \frac{\text{Volt}}{\text{sec.} \times \text{amp./se.}} = \text{ohm.} (\Omega)$$

Q. A 44 mH inductor is connected to 220 V, 50 Hz a.c. supply. Determine the r.m.s. value of current in the circuit.

Sol. Given that $L = 44\,mH = 44 \times 10^{-3}\,H$

$E_{rms} = 220\,V$, $f = 50\,Hz$

so $X_L = \omega L = 2\pi f L$

$$X_L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3} = 13.83 \text{ ohm } (\Omega)$$

$$I_{rms} = \frac{E_{rms}}{X_L} = \frac{220}{13.83} = 15.9\,A$$

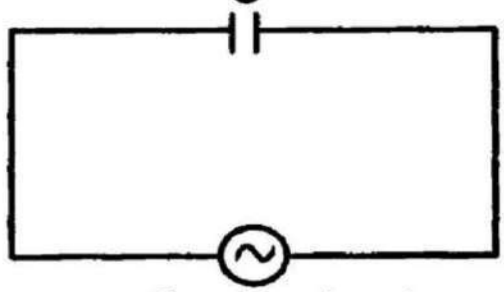
A.C. Circuit Containing Capacitance Only

Let a source of alternating e.m.f. is connected to a capacitor only of capacitance C. Suppose the alternating e.m.f. supplied is -

$$E = E_0 \sin \omega t$$

The current flowing in the circuit transfers charge to the plates of the capacitor. This produces a potential difference between the plates.

C



$E = E_0 \sin \omega t$

At any instant t, suppose q is the charge on the capacitor. So the potential difference across the plates of capacitor is

$$V = q/C$$

At every instant, the potential difference V must be equal to the e.m.f. applied so -

$$V = E$$

$$\frac{q}{C} = E_0 \sin \omega t$$

or $$q = C E_0 \sin \omega t$$

If I is the instantaneous value of current in the circuit at instant t, then -

$$I = \frac{dq}{dt} = \frac{d}{dt}(C E_0 \sin \omega t)$$

$$I = C E_0 (\cos \omega t) \cdot \omega$$

$$I = \frac{E_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The current will be maximum $I = I_0$, when $\sin\left(\omega t + \frac{\pi}{2}\right)$ = maximum = 1

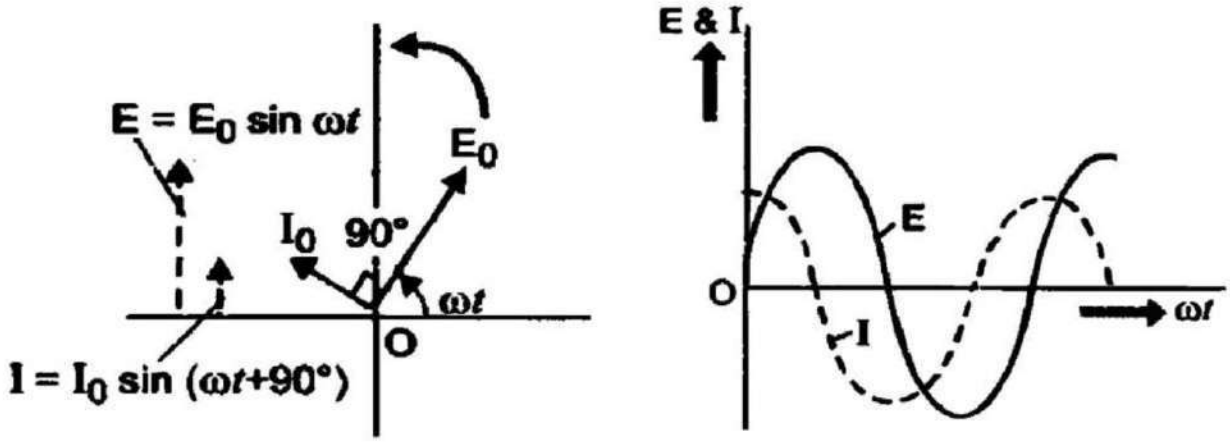
so $$I_0 = \frac{E_0}{1/\omega C}$$

and $$\boxed{I = I_0 \sin(\omega t + \pi/2)}$$

We find that in an a.c. circuit containing c only alternating current I Leads the alternating e.m.f. by a phase angle of 90°

Phasor Diagram -

The phasor I_0 is turned anticlockwise through 90° from the direction of phasor E_0. There projections on y-axis gives the instantaneous values of E and I.



Here $1/\omega c$ represents effective resistance offered by the capacitor. This is called capacitive reactance and is denoted by X_C.

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

★ In dc circuit $f = 0$ so $X_C = \infty$
So a Capacitor will block d.c.

★ SI unit of X_C is ohm. (Ω)

Note-

(i) R can reduce a.c. as well as d.c.
L cannot reduce d.c. It can reduce only a.c.
C blocks d.c. but allows a.c. to pass through.

(ii) Through R, alternating current is in phase with alternating voltage.
Through L, alternating current Lags behind the alternating voltage by a phase angle 90°.
Through C, alternating current leads the alternating voltage by a phase angle of 90°.

Q. (CBSE 2009) A 15 μF capacitor is connected to 220V, 50 Hz source, Find the peak current.

Sol. Here $C = 15\,\mu F = 15 \times 10^{-6}\,F$

$E_{rms} = 220\,V$, $f = 50\,Hz$.

so $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}}$

$X_C = 212.1\,\Omega$

$I_{rms} = \frac{E_{rms}}{X_C} = \frac{220}{212.1} = 1.037\,A$

and $I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 1.037$

so $I_0 = 1.47\,A$

A.C. Circuit Containing Resistance, Inductance and Capacitance in Series (RLC Circuit)

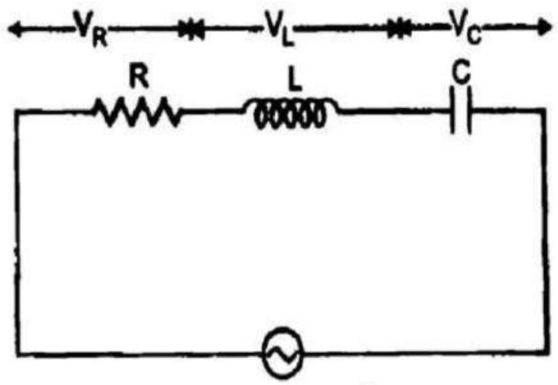
Let R, L, C are connected in series to a source of alternating e.m.f. so the current at any instant through the three elements has the same amplitude and phase.

$$\boxed{I = I_0 \sin\omega t}$$

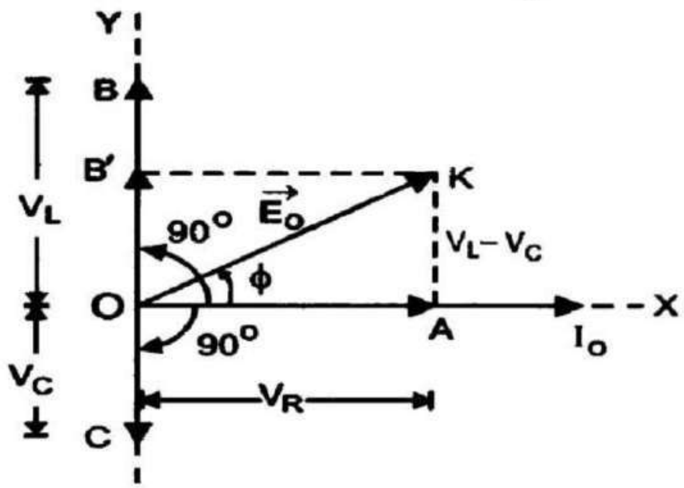
(i) The maximum voltage across R is

$$V_R = I_0 R$$

In figure, current Phasor I_0 is represented along OX. As V_R is in phase with current, so it is represented by the vector $\overrightarrow{OA}$, along OX.



$I = I_0 \sin\omega t$



(ii) The maximum voltage across L is $V_L = I_0 X_L$. As voltage across the inductor leads the current by 90°, so it is represented by $\overrightarrow{OB}$ along OY, 90° ahead of I_0.

(iii) The maximum value across C is $V_C = I_0 X_C$. As the voltage across the capacitor Lags behind the alternating current by 90°, so it is represented by $\overrightarrow{OC}$ rotated clockwise through 90° from the direction of I_0.

As the voltage across L and C have a phase difference of 180°, so net reactive voltage is $(V_L - V_C)$, assuming that $V_L > V_C$. In figure it is represented by $\overrightarrow{OB'}$.

The resultant of $\vec{OA}$ and $\vec{OB'}$ is the diagonal $\vec{OK}$.

Hence the vector sum of $\vec{V_R}$, $\vec{V_L}$ and $\vec{V_C}$ is phasor $\vec{E_0}$ represented by $\vec{OK}$, making an angle ϕ with current phasor $\vec{I_0}$.

$$\text{so} \quad OK = \sqrt{OA^2 + OB'^2}$$

$$\therefore \quad E_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$E_0 = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2}$$

$$\text{or} \quad E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

The total effective resistance of RLC circuit is called Impedance of the circuit. It is represented by Z, where,

$$Z = \frac{E_0}{I_0}$$

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

* So in an a.c. circuit containing R, L, C, the voltage leads the current by a phase angle ϕ, where

$$\tan\phi = \frac{OB'}{OA} = \frac{V_L - V_C}{V_R} = \frac{I_0 X_L - I_0 X_C}{I_0 R}$$

$$\text{or} \quad \boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

So the alternating e.m.f. in an RLC circuit is represented by -

$$\boxed{E = E_0 \sin(\omega t + \phi)}$$

There may be three cases -

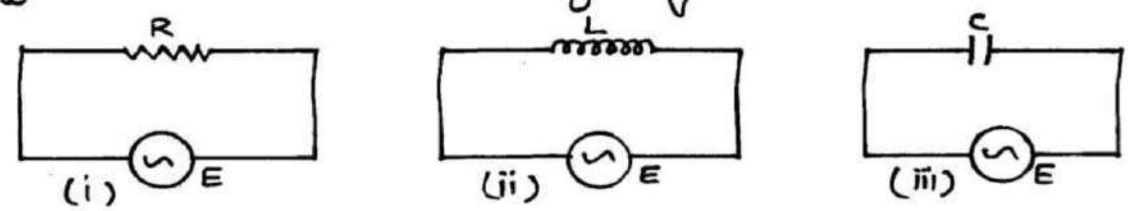
(i) when $X_L = X_C$, then $\tan\phi = 0$

so $\boxed{\phi = 0^\circ}$

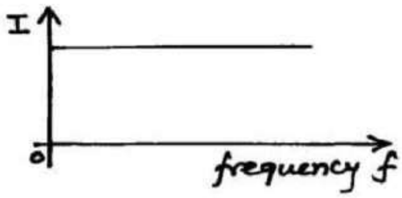
(ii) when $X_L > X_C$, then $\tan\phi$ is positive.
So ϕ is positive.
Hence voltage leads the current by a phase angle ϕ.

(iii) when $X_L < X_C$, then $\tan\phi$ is negative.
So ϕ is negative.
Hence voltage Lags behind the current by a phase angle ϕ.

Q. (Delhi 2011) Three electrical circuits having AC sources of variable frequency are shown in figures. Initially the current flowing in each of these is same. If the frequency of the applied AC source is increased how will the current flowing in these circuits be affected? Give reason for your answer.



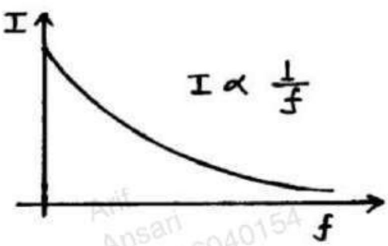
Sol. (i) There will not be any effect in the current on changing the frequency of AC source.



(ii) With the increase of freq. of AC source, inductive reactance increases as -

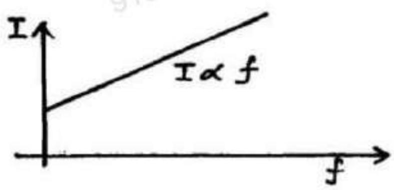
$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi f L}$$

so current decreases with the increase of frequency.



(iii) The capacitive reactance decreases with increase in frequency so the current will increase.

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{1/2\pi f C}$$



Q. A resistor of 400 Ω, an inductor of $\frac{5}{\pi}$ H and a capacitor of $\frac{50}{\pi}$ μF are connected in series across a source of alternating voltage of 140 sin 100πt V. Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltages more than the source voltages? If yes, resolve the paradox.

CBSE 2010

Sol. Applied Voltage $V = 140 \sin 100\pi t$

$$C = \frac{50}{\pi}\,\mu F, \quad C = \frac{5}{\pi} H, \quad R = 400\,\Omega$$

Comparing with $V = V_0 \sin\omega t$

$$V_0 = 140\,V, \quad \omega = 100\pi$$

Inductive reactance $X_L = \omega L$

$$X_L = 100\pi \times \frac{5}{\pi} = 500\,\Omega$$

Capacitive reactance $X_C = 1/\omega C$

$$X_C = \frac{1}{100\pi \times \frac{50}{\pi} \times 10^{-6}} = 200\,\Omega$$

Impedance of the circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$Z = \sqrt{(400)^2 + (500-200)^2} = 500\,\Omega$$

Maximum current in the circuit

$$I_0 = \frac{V_0}{Z} = \frac{140}{500}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{140}{500 \times \sqrt{2}} = 0.2\,A$$

Vrms across resistor $V_R = I_{rms} R$

$$V_R = 0.2 \times 400 = 80\,V$$

Vrms across inductor $V_L = I_{rms} X_L$

$$V_L = 0.2 \times 500 = 100\,V$$

Vrms across Capacitor $V_C = I_{rms} X_C$

$$V_C = 0.2 \times 200 = 40\,V$$

* Here $V \neq V_R + V_L + V_C$

Because V_L and V_C and V_R are not in same phase instead $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

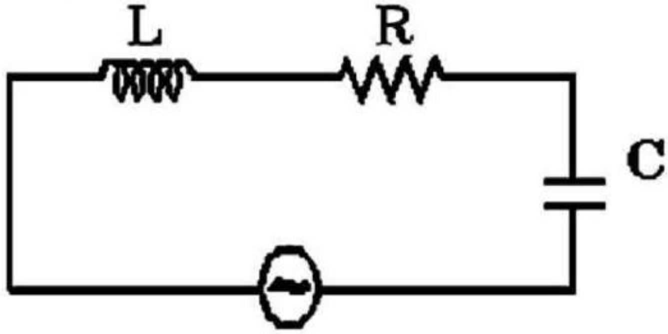
Electrical Resonance

The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the natural frequency of oscillation of the system.

If such a system is driven by an energy source, whose frequency is equal to the natural frequency of the system, then the amplitude of oscillations becomes large and resonance is said to occur.

Series Resonance Circuit

A circuit in which inductance L, capacitance C and resistance R are connected in series, and the circuit admits maximum current corresponding to a given frequency of a.c., is called series resonance circuit.



At very low frequencies, inductive reactance $X_L = \omega L$ is negligible but capacitive reactance $X_C = 1/\omega C$ is very high.

As the frequency of alternating e.m.f. applied to the circuit is increased, X_L goes on increasing and X_C goes on decreasing.

For a particular value of ω (say ω_r)

$$X_L = X_C$$

$$\omega_r L = \frac{1}{\omega_r C}$$

or $$\boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

and $\quad 2\pi f_r = \frac{1}{\sqrt{LC}}$

so $\quad \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$

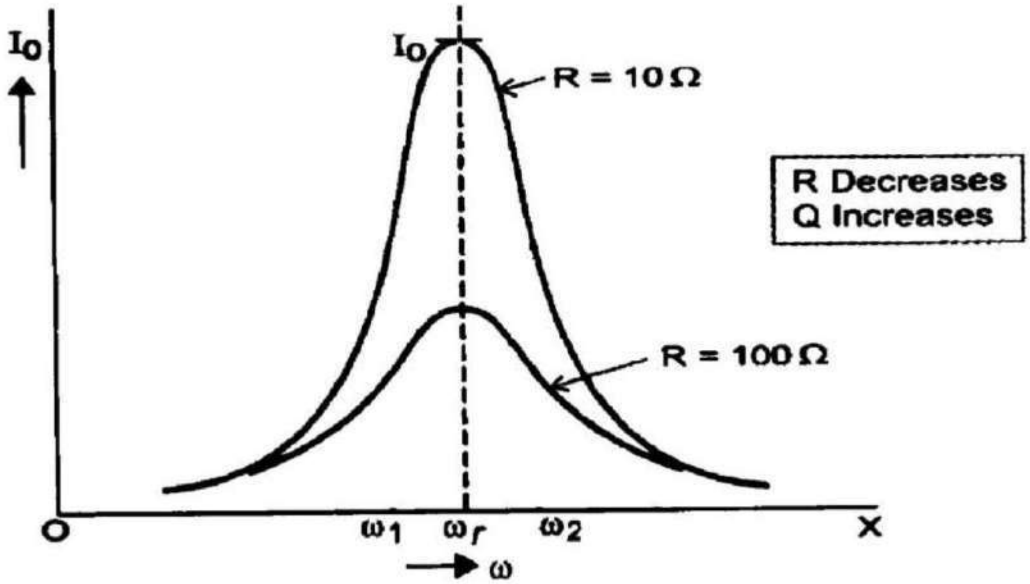
At this particular frequency f_r as $X_L = X_C$ so

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + 0} = R \quad (\text{minimum})$$

So Here impedance of RLC circuit is minimum and hence the current $I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$ becomes maximum.

This frequency is called series resonance frequency.



The variation of circuit peak current with the changing frequency of applied voltage is shown in figure.

(i) It is clear that for frequencies greater than or less than ω_r the values of peak current are less than the maximum value (I_0)

(ii) at $\omega = \omega_r$ the value of peak current is maximum (I_0).

Applications -

Series resonance circuit is used in radio and T.V. receiver sets.

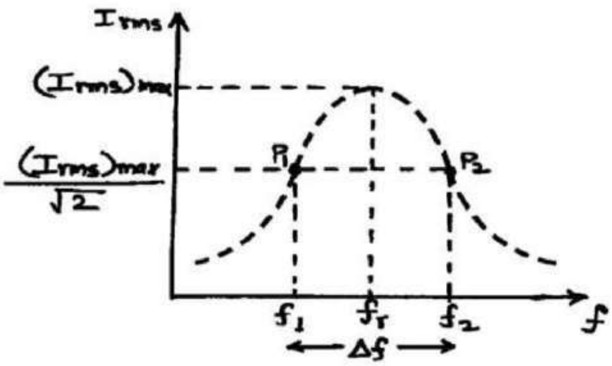
The antenna of a radio/ T.V. intercepts signals from many broadcasting stations. To receive one particular radio station / T.V. channel, we tune our receiver set by changing the capacitance of a capacitor in the tuning circuit of the set such that resonance frequency of the circuit becomes equal to the frequency of the desired station.

Therefore the resonance occurs and the amplitude of current with the frequency of the signal from the desired station becomes maximum and it is received in our set.

Half power frequencies

In RLC circuit the current at resonance is maximum. At both sides of the resonating frequency the current decreases with increase or decrease in frequency.

The power of a circuit is directly proportional to the square of the current ($P \propto I^2$). If we want to decrease the power upto half then the current should be $\frac{1}{\sqrt{2}}$ times of its initial value.



In the both sides of f_r there are two frequencies f_1 and f_2 at which the current is $(I_{rms})_{max}/\sqrt{2}$, hence the power at these frequencies f_1 and f_2 becomes half.

So these frequencies are called "half power frequencies". On the graph P_1 and P_2 are called the half power points.

Band Width -

The difference of half power frequencies is called band width. It is represented by β or Δf.

$$\boxed{\text{Band width } \beta \text{ or } \Delta f = f_2 - f_1}$$

If the effective current at half power points is I_{rms} then,

$$I_{rms}^2 R = \frac{1}{2}(I_{rms})_{max}^2 R$$

$$I_{rms} = \frac{(I_{rms})_{max}}{\sqrt{2}} = 0.707 (I_{rms})_{max}$$

So the current at half power points P_1 and P_2 becomes $\frac{1}{\sqrt{2}}$ or 0.707 times of the current at resonance.

so that by putting the values in above equation

$$\frac{V_{rms}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{V_{rms}}{\sqrt{2}R}$$

or
$$R^2 + (\omega L - \frac{1}{\omega C})^2 = 2R^2$$

or
$$\omega L - \frac{1}{\omega C} = \pm R$$

For point P_1 , $\omega_1 L - \frac{1}{\omega_1 C} = -R$ — ①

For Point P_2 , $\omega_2 L - \frac{1}{\omega_2 C} = +R$ — ②

by adding equation ① and ② we get -

$$(\omega_1 + \omega_2) L - \frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \times \frac{1}{C} = 0$$

or
$$\omega_1 \omega_2 = \frac{1}{LC}$$

by subtracting equation ② by ① we get -

$$(\omega_2 - \omega_1) L + \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R$$

or
$$(\omega_2 - \omega_1) L + \frac{\omega_2 - \omega_1}{C\omega_1 \omega_2} = 2R$$

$$(\omega_2-\omega_1)L + \frac{\omega_2-\omega_1}{C\times\frac{1}{LC}} = 2R$$

$$(\omega_2-\omega_1)\times 2L = 2R$$

$$\omega_2-\omega_1 = \frac{R}{L}$$

The difference of frequencies corresponding to the half power points is called bandwidth

$$\boxed{f_2 - f_1 = \frac{R}{2\pi L} = \text{Band width}}$$

[redacted]

Q - Factor of Resonance Circuit

The characteristic of a series resonant circuit is determined by the Q factor or Quality factor of the circuit.
It defines sharpness of tuning at resonance.

The Q-factor of a series resonant circuit is defined as the ratio of the voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the voltage applied across R.

$$\boxed{Q = \frac{\text{Voltage across L or C}}{\text{applied Voltage (= Voltage across R)}}}$$

$$Q = \frac{V_L}{V_R} \quad \text{or} \quad Q = \frac{V_C}{V_R}$$

$$Q = \frac{(\omega_r L) I}{R I} = \frac{\omega_r L}{R}$$

using $\omega_r = \frac{1}{\sqrt{LC}}$,

$$Q = \frac{L}{R} \frac{1}{\sqrt{LC}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$Q = \frac{(1/\omega_r C) I}{R I} = \frac{1}{R C \omega_r}$$

using $\omega_r = \frac{1}{\sqrt{LC}}$,

$$Q = \frac{1}{RC}\sqrt{LC}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

- ★ As R is increased, Q factor of the circuit decreases.
- ★ The values of Q usually vary from 10 to 100.
- ★ The electronic circuits with high Q values would respond to a very narrow range of frequencies and sharper is the resonance.

Q. In a series L-C-R circuit, the voltages across an inductor, a capacitor and a resistor are 30V, 30V and 60V respectively. What is the phase difference between the applied voltage and the current in the circuit?

CBSE 2007

Sol. Given that - $V_L = 30V$, $V_C = 30V$, $V_R = 60V$

Here we can see that $V_L = V_C$, Hence the circuit is in resonance, so there will be no phase difference between the applied voltage and current.

Q. A radio can tune over the frequency range of a portion of MW broadcast band (800 KHz to 1200 KHz). If its LC circuit has an effective inductance of 200 μH, then what must be the range of its variable capacitor?

Sol Given that -

$$f_1 = 800 \text{ KHz} = 8 \times 10^5 \text{ Hz}$$

$$f_2 = 1200 \text{ KHz} = 12 \times 10^5 \text{ Hz}$$

$$L = 200 \text{ μH} = 200 \times 10^{-6} \text{ H}$$

Capacity range between C_1 and C_2 -

From $$f_1 = \frac{1}{2\pi\sqrt{LC_1}}$$

So $$C_1 = \frac{1}{4\pi^2 L f_1^2} = \frac{1}{4 \times (3.14)^2 \times 2 \times 10^{-4} \times (8 \times 10^5)^2}$$

$$C_1 = 197.7 \times 10^{-12} \text{ F} = 197.7 \text{ PF}$$

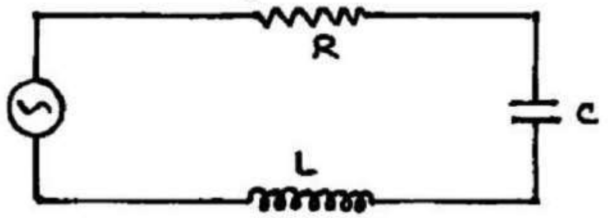
Similarly, $$C_2 = \frac{1}{4\pi^2 L f_2^2}$$

$$C_2 = \frac{1}{4 \times (3.14)^2 \times 2 \times 10^{-4} \, (12 \times 10^5)^2}$$

$$C_2 = 87.8 \times 10^{-12} \text{ F} = 87.8 \text{ PF}$$

Q. Delhi 2012

The figure shows a series L-C-R circuit with $L = 10\,H$, $C = 40\,\mu F$, $R = 60\,\Omega$, connected to a variable frequency 240 V source, calculate-



(a) The angular frequency of the source which drives the circuit at resonance.

(b) The current at the resonating frequency.

(c) the rms potential drop across the inductor at resonance.

Sol. Given that- $L = 10\,H$, $C = 40\,\mu F$, $R = 60\,\Omega$

$V_{rms} = 240\,V$

(a) Resonating angular frequency

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 40 \times 10^{-6}}}$$

$$\omega_r = 50 \text{ rad/s.}$$

(b) Current at resonating frequency

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} \quad (\because \text{at resonance } Z = R)$$

$$I_{rms} = \frac{240}{60} = 4\,A$$

(c) Inductive reactance $X_L = \omega L$

At resonance $X_L = \omega_r L$

$$X_L = 50 \times 10 = 500\,\Omega$$

Potential drop across inductor

$$V_{rms} = I_{rms} \times X_L$$

$$= 4 \times 500$$

$$V_{rms} = 2000\,V.$$

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Q. CBSE 2018 When a given coil is connected to a 200 V DC source, 2A current flows and when the same coil is connected to 200 V, 50 Hz AC source, only 1A current flows in the circuit.

(a) Explain why the current decreases in the later case?

(b) Calculate the self inductance of the coil used.

Sol. (a) In DC circuit -

$X_L = 2\pi f L = 0 \qquad (\because f = 0)$

$\therefore$ The impedance of the coil $Z = \sqrt{R^2 + X_L^2}$

In DC, $Z = R$ as $X_L = 0$

But in AC, $Z = \sqrt{R^2 + X_L^2} > R$

because X_L have certain value in AC.

As $I \propto \frac{1}{Z}$ hence in DC, more current flows than AC.

(b) In DC circuit -

$$R = \frac{V}{I} = \frac{200}{2} = 100\,\Omega$$

In AC circuit -

$$Z = \frac{V}{I} = \frac{200}{1} = 200\,\Omega$$

As $R^2 + X_L^2 = Z^2$

So $X_L^2 = Z^2 - R^2$

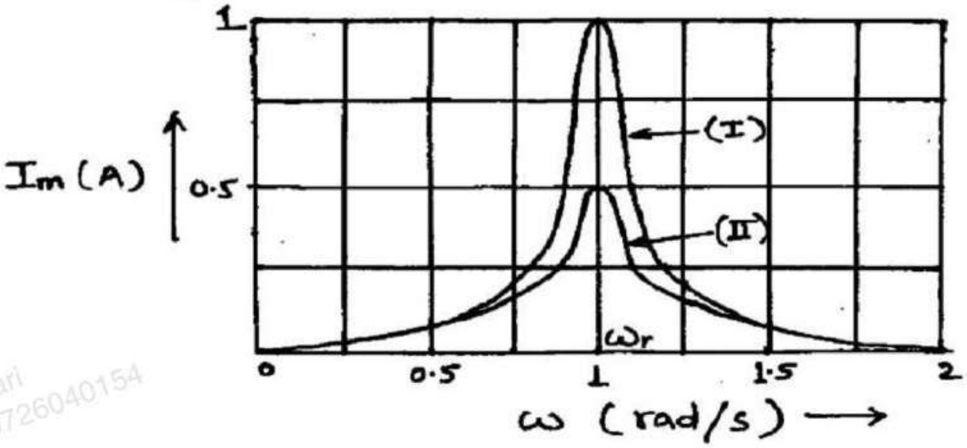
$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(200)^2 - (100)^2} = 100\sqrt{3}\,\Omega$$

$\Rightarrow \quad X_L = 2\pi f L = 100\sqrt{3}$

$\therefore$ Inductance $L = \dfrac{100\sqrt{3}}{2\pi f}$

$$L = \frac{100\sqrt{3}}{2 \times 3.14 \times 50} = 0.55\ H$$

Q. *Delhi 2009* The graphs shown here, depict the variation of current I_{rms} angular frequency (ω) for two different series L-C-R circuits.



observe the graphs carefully.

(i) State the relation between L and C values of the two circuits, when the current in the two circuits is maximum.

(ii) Indicate the circuit for which
(a) power factor is higher
(b) Quality factor (Q) is Larger.

<u>Sol.</u> (i) Since resonant angular frequency ω_r is same for both the graphs-

$$\therefore \quad (\omega_r)_1 = (\omega_r)_2$$

$$\frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}} \Rightarrow \frac{L_1}{L_2} = \frac{C_2}{C_1}$$

(ii) (a) In graph (I), current I_{m_1} is greater than graph (II) current I_{m_2}.

i.e. $I_{m_1} > I_{m_2} \Rightarrow R_1 < R_2$

Power factor of circuit represented by graph (II) has higher power factor as $\cos\phi \propto R$.

(b) $\because Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ for two graphs

$Q \propto \frac{1}{R}$ $\quad \because R_1 < R_2$

$\therefore Q_1 > Q_2$

Quality factor of graph (I) is higher than the graph (II).

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Q. An inductor 200 mH, Capacitor 500 μF and a resistor 10 Ω are connected in series with a 100 V, variable frequency AC source. Calculate

Delhi 2018

(a) the frequency at which the power factor of the circuit is unity.

(b) Current amplitude at this frequency

(c) Q- factor.

Sol. (a) When power factor $\cos\phi = 1$, hence $\phi = 0$ and the L-C-R circuit is in resonance.

$\therefore$ Resonating frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_0 = \frac{1}{\sqrt{200 \times 10^{-3} \times 500 \times 10^{-6}}} = 100\ s^{-1}$$

(b) $$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} \quad (\because \text{ at resonance } Z = R)$$

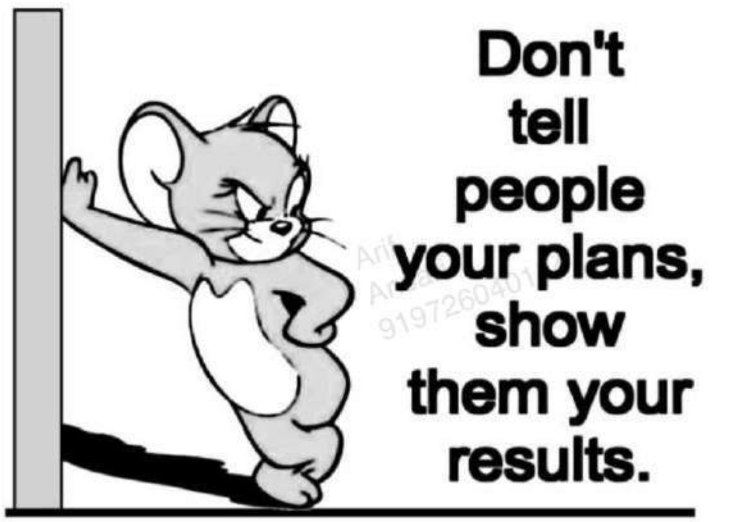
$$I_{rms} = \frac{100}{10} = 10\ A$$

Amplitude of maximum current $I_0 = \sqrt{2}\, I_{rms}$

$$I_0 = 10\sqrt{2}\ A$$

(c) $$Q.\ \text{factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{500 \times 10^{-6}}} = 20$$



Average Power in RLC Circuit

Let the alternating e.m.f. applied to an RLC circuit is -

$$E = E_0 \sin\omega t$$

If alternating current developed lags behind the applied e.m.f. by a phase angle ϕ, then -

$$I = I_0 \sin(\omega t - \phi)$$

Power at instant t, $\frac{dw}{dt} = EI$

So $\frac{dW}{dt} = E_0 \sin\omega t \times I_0 \sin(\omega t - \phi)$

$$= E_0 I_0 \sin\omega t (\sin\omega t \cos\phi - \cos\omega t \sin\phi)$$

$$= E_0 I_0 \sin^2\omega t \cos\phi - E_0 I_0 \sin\omega t \cos\omega t \sin\phi$$

$$= E_0 I_0 \sin^2\omega t \cos\phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi$$

If this instantaneous power is assumed to remain constant for a small time dt, then small amount of work done in this time is -

$$dW = \left(E_0 I_0 \sin^2\omega t \cos\phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi\right) dt$$

Total work done over a complete cycle is -

$$W = \int_0^T E_0 I_0 \sin^2\omega t \cos\phi \, dt - \int_0^T \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi \, dt$$

$$W = E_0 I_0 \cos\phi \int_0^T \sin^2\omega t \, dt - \frac{E_0 I_0}{2} \sin\phi \int_0^T \sin 2\omega t \, dt$$

As $\int_0^T \sin^2\omega t \, dt = \frac{T}{2}$ and $\int_0^T \sin\omega t \, dt = 0$

$\therefore$ $$W = E_0 I_0 \cos\phi \times \frac{T}{2}$$

$\therefore$ Average power in the RLC circuit over a complete cycle is -

$$P_{av} = \frac{W}{T}$$

So $$P_{av} = \frac{E_0 I_0 \cos\phi}{T} \times \frac{T}{2}$$

$$P_{av} = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi$$

$$\boxed{P_{av} = E_{rms} \, I_{rms} \cos\phi}$$

Hence average power over a complete cycle in an inductive circuit is the product of r.m.s. e.m.f. and r.m.s. current and cosine of the phase angle between the voltage and current.

* The above relation is applicable to all a.c. circuits.

$\cos\phi$ and Z will have appropriate values for different circuits.

(i) Only R circuit -

$Z = R$ and $\cos\phi = 1$

(ii) Only L circuit -

$Z = X_L$ and $\cos\phi = 0$

(iii) Only C circuit -

$Z = X_C$ and $\cos\phi = 0$

(iv) In RLC circuit -

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\cos\phi = \frac{R}{Z}$

(v) In RL circuit -

$Z = \sqrt{R^2 + X_L^2}$ and $\cos\phi = \frac{R}{Z}$

(vi) In RC circuit -

$Z = \sqrt{R^2 + X_C^2}$ and $\cos\phi = \frac{R}{Z}$

(vii) In LC circuit -

$Z = \sqrt{(X_L - X_C)^2} = X_L - X_C$, $\cos\phi = 0$, and $\phi = 90^\circ$

Power Factor of an A.C. Circuit-

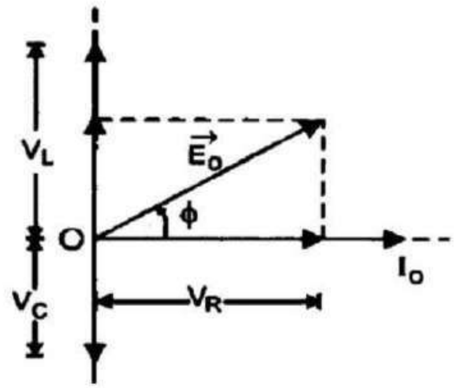
We Know that the average power in RLC Circuit is given by -

$$\boxed{P = E_{rms} I_{rms} \cos\phi}$$

Here P is called true power, ($E_{rms} I_{rms}$) is called apparent power or virtual power and $\cos\phi$ is called power factor of the circuit.

$$\boxed{\text{Power Factor} (\cos\phi) = \frac{\text{True Power} (P)}{\text{apparent Power}}}$$

From diagram-



$$\cos\phi = \frac{V_R}{E_0}$$

$$= \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$

$$\boxed{\text{Power factor} = \cos\phi = \frac{R}{Z}}$$

(i) In a non-inductive circuit, $X_L = X_C$

$\therefore$ Power factor $= \cos\phi = \frac{R}{\sqrt{R^2}} = \frac{R}{R} = 1$

$$\phi = 0^\circ$$

This is the maximum value of power factor.

(ii) In a Pure inductor or an ideal capacitor

$$\phi = 90^\circ$$

$\therefore$ Power Factor $= \cos\phi = \cos 90^\circ = 0$

So average power consumed in a pure inductor or ideal capacitor

$$P = E_{rms} I_{rms} \cos\phi = 0$$

In actual practice we do not have ideal inductor or ideal capacitor. Therefore, there does occur some dissipation of energy.

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Q. An inductor and a bulb are connected in series to an AC source of 12V, 50 Hz. 2A Current flows in the circuit and the phase angle between voltage and current is $\pi/4$ rad. Calculate the impedance and inductance of the circuit.

CBSE 2019

Sol. Given that - $V_{rms} = 12V$, $f = 50\,Hz$

$\phi = \pi/4$, $I_{rms} = 2A$

$\therefore$ Impedance $Z = \dfrac{V_{rms}}{I_{rms}} = \dfrac{12}{2} = 6\,\Omega$

Also, power factor $\cos\phi = \dfrac{R}{Z}$

$R = Z\cos\phi$

$R = 6 \times \cos\dfrac{\pi}{4} = 6 \times \dfrac{1}{\sqrt{2}} = 3\sqrt{2}\,\Omega$

In L-R circuit $Z^2 = R^2 + X_L^2$

so $X_L^2 = Z^2 - R^2$

$X_L = \sqrt{(6)^2 - (3\sqrt{2})^2} = 3\sqrt{2}\,\Omega$

But $X_L = 2\pi f L = 3\sqrt{2}$

so $L = \dfrac{3\sqrt{2}}{2\pi f} = \dfrac{3\sqrt{2}}{2 \times 3.14 \times 50} = 13.5\,mH$

Q. A capacitor and a resistor are connected in series with an AC source. If the potential difference across C, R are 120 V, 90 V respectively and if the rms current of the circuit is 3A, calculate the (a) Impedance,
(b) Power factor of the circuit.

CBSE 2016

Sol. Given that - $V_C = 120V$, $V_R = 90V$, I_{rms} 3A

(a) In C-R series AC Circuit

$V_{rms} = \sqrt{V_R^2 + V_C^2}$

$V_{rms} = \sqrt{(120)^2 + (90)^2} = 150\,V$

$I_{rms} = \dfrac{V_{rms}}{Z}$ so $Z = \dfrac{V_{rms}}{I_{rms}}$

$$I_{rms} = \frac{V_{rms}}{Z} \qquad \text{So} \qquad Z = \frac{V_{rms}}{I_{rms}}$$

$$Z = \frac{150}{3} = 50\ \Omega$$

(b) $$V_R = R \quad \Rightarrow \quad R = \frac{V_R}{I_{rms}}$$

$$R = \frac{90}{3} = 30\ \Omega$$

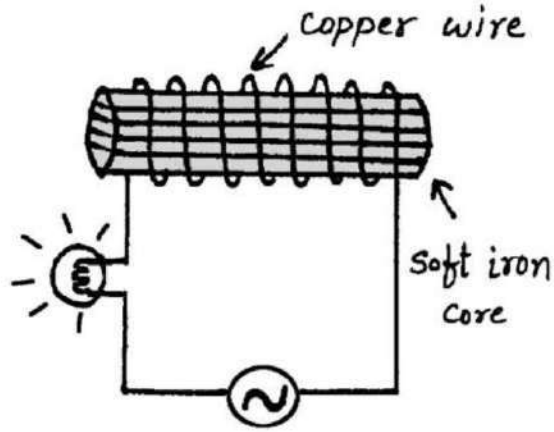
$\therefore$ Power factor $(\cos\phi) = \frac{R}{Z}$

Power factor $= \frac{30}{50} = 0.6$

CHOKE COIL

Choke coil is simply a coil having a large inductance but a small resistance.

Construction – Choke coil is made by a thick copper wire to reduce its resistance. It is wounded on a plated soft iron core to increase its self inductance.



Principle –

At most places, the household electric power is supplied at 220 V, 50 Hz. If such a source is directly connected to a mercury tube, the tube will be damaged.

To reduce the current, a choke coil is connected in series with the tube.

Power Consumed in an A.C. circuit is $P = E_{rms} I_{rms} \cos\phi$

For pure inductor $\phi = \frac{\pi}{2}$

So that $P = 0$

Hence the advantage of using choke coil to reduce the voltage is that an inductor does not consume power. Hence we do not lose electric energy in the form of heat.

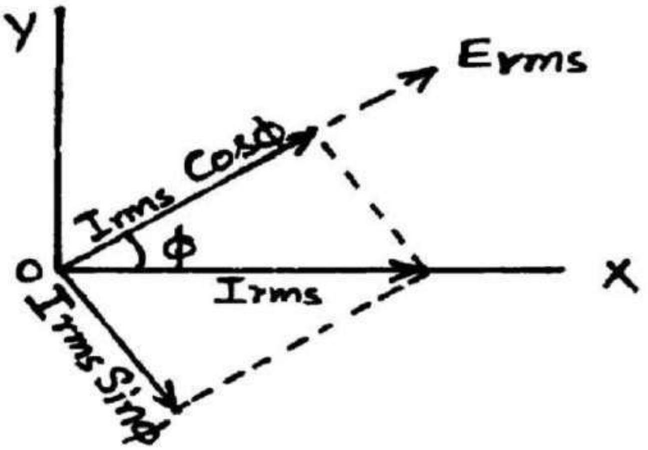
Wattless Current or Idle Current

The current which consumes no power for its maintenance in the circuit is called wattless current.

We know that average power over a complete cycle in an inductive circuit is-

$$\boxed{P = E_{rms} I_{rms} \cos\phi}$$

In a particular circuit, suppose E_{rms} leads I_{rms} by phase angle ϕ, as shown in figure.



I_{rms} Can be resolved into two rectangular components

As phase angle between E_{rms} and $I_{rms}\cos\phi$ is zero.

$\therefore$ average power consumed per cycle in the circuit due to component $I_{rms}\cos\phi$ is -

$$P_{av} = E_{rms}(I_{rms}\cos\phi)\cos 0^\circ = E_{rms} I_{rms} \cos\phi$$

Again as phase angle between E_{rms} and $I_{rms}\sin\phi$ is $\pi/2$, So average power consumed per cycle in the circuit due to component $I_{rms}\sin\phi$ is -

$$P'_{av} = E_{rms}(I_{rms}\sin\phi)\cos\frac{\pi}{2} = 0$$

The Component $I_{rms}\sin\phi$ makes no Contribution to the consumption of power in the a.c. circuit.

That is why this Component **$I_{rms}\sin\phi$** is called the idle component or **wattless** component of a.c. and the component **$I_{rms}\cos\phi$** is called **wattfull** component of a.c.

A.C. Generator or A.C. Dynamo

An a.c. generator / dynamo is a machine which produces alternating current energy from mechanical energy.

It is most important applications of the phenomenon of electromagnetic induction. The generator was designed originally by a Yugoslav scientist, Nikola Tesla.

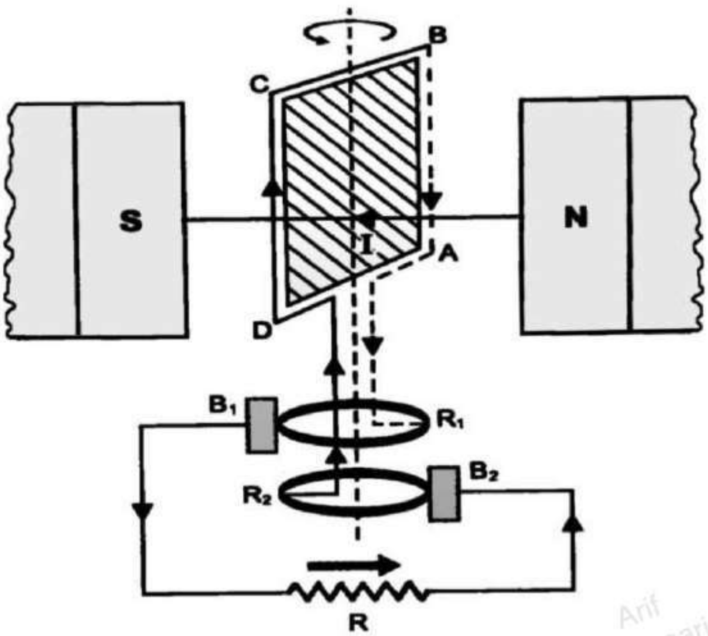
Principle -

An a.c. generator / dynamo is based on the phenomena of electromagnetic induction. Whenever the amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the coil.

The essential parts of an a.c. generator are-

(i) Armature -

ABCD is a rectangular armature coil. It consist of a large number of turns of insulated copper wire wound over a laminated soft iron core. The coil can be rotated about the central axis.



(ii) Field Magnets -

N and S are the pole pieces of a strong electromagnet in which the armature coil is rotated. Axis of rotation is perpendicular to the magnetic field lines.

(iii) **Slip Rings -** R_1 and R_2 are two hollow metallic rings, to which two ends of armature coil are connected. These rings rotate with the rotation of the coil.

(iv) **Brushes -** B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in light contact with R_1 and R_2 respectively. The purpose of brushes is to pass on current from the armature coil to the external load resistance R.

Working

As the armature coil is rotated in the magnetic field, angle θ between the field and normal to the coil changes continuously. Therefore magnetic flux linked with the coil changes. And an e.m.f. is induced in the coil.

To start with, suppose the coil is perpendicular to the plane of the paper in which magnetic field is applied, with AB at front and CD at the back. The amount of flux linked with coil in this position is maximum.

As the coil is rotated anticlockwise, AB moves inwards and CD moves outwards. The amount of magnetic flux linked with the coil changes.

According to Fleming's right hand rule, current induced in AB is from A to B and in CD, it is from C to D. In the external circuit, current flows from B_2 to B_1.

After half the rotation of the coil, AB is at the back and CD is at the front (figure (b)). Therefore on rotating further, AB moves outwards and CD moves inwards.

The current induced in AB is from B to A and in CD, it is from D to C. Through external circuit, current flows from B_1 to B_2.

Induced current in the external circuit changes direction after every half rotation of the coil. Hence the current induced in coil is alternating in nature.

Magnitude of induced e.m.f.

Magnetic flux linked with the coil

$$\phi = N(\vec{B}\cdot\vec{A}) = NBA\cos\theta$$

or $$\phi = NBA\cos\omega t \quad — (1)$$

where ω is angular velocity of the coil.

As the coil is rotated, θ changes, therefore magnetic flux ϕ linked with the coil changes and hence an e.m.f. is induced in the coil.

At this instant t, if e is the e.m.f. induced in the coil, then -

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t)$$

$$e = -NBA\frac{d}{dt}(\cos\omega t) = -NBA(-\sin\omega t)\omega$$

or $$e = NBA\omega\sin\omega t \quad — (2)$$

The induced e.m.f. will be maximum, when

$$\sin\omega t = \text{maximum} = 1$$

$$e_{max} = e_0 = NBA\omega \times 1 \quad — (3)$$

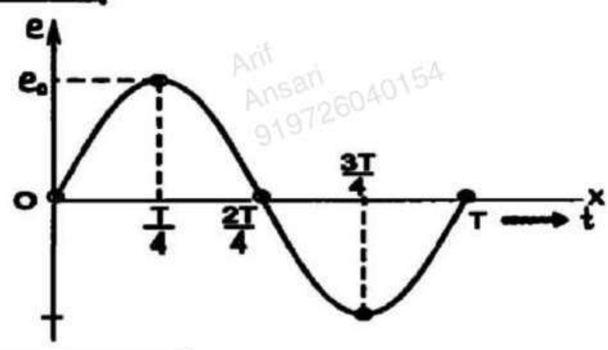
by equation (2) we get -

$$\boxed{e = e_0 \sin\omega t}$$

$$I = \frac{e}{R} = \frac{e_0}{R}\sin\omega t$$

$$I_0 = \frac{e_0}{R}$$

$$\boxed{I = I_0 \sin\omega t}$$



Transformer

A transformer is an electrical device which is used for changing the a.c. voltages.

A transformer which increases the a.c. voltages is called step up transformer and a transformer which decreases the a.c. voltages is called a step down transformer.

Principle -

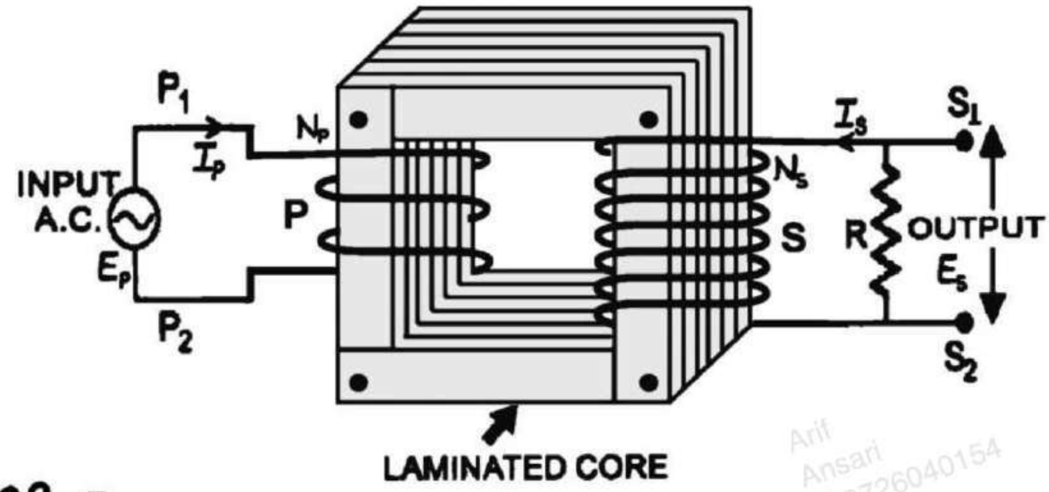
A transformer is based on the principle of mutual induction i.e. whenever the amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the neighbouring coil.

Construction -

A transformer consist of a soft iron core made of laminated sheets, well insulated from each other.

Two coils P_1P_2 (Primary Coil) and S_1S_2 (secondary coil) are wound on the same core, but are well insulated from each other. Note that both the coils are also insulated from the core.

The source of alternating e.m.f. (to be transformed) is connected to the primary coil P_1P_2 and a load resistance R is connected to the secondary coil S_1S_2.



Working -

Let the alternating e.m.f. supplied by the a.c. source connected to primary is -

$$E_P = E_0 \sin\omega t$$

The alternating primary current induces an alternating magnetic flux ϕ_B in the iron core. Because the core extends through the turns of secondary, so the induced flux also extends through the secondary winding.

According to the Faraday's Law of electromagnetic induction, the induced e.m.f. per turn (E_{turn}) is same for both primary & secondary winding.

$$E_{turn} = \frac{d\phi_B}{dt} = \frac{E_P}{N_P} = \frac{E_S}{N_S}$$

Here n_p, n_s represents the total number of turns in the primary and secondary coils respectively.

$$\boxed{\frac{E_s}{E_p} = \frac{N_s}{N_p}}$$

As we assume that no energy is lost along the way, conservation of energy requires that-

Input Power = Output Power

$$I_P E_P = I_s E_s \quad \therefore \quad \boxed{\frac{I_P}{I_s} = \frac{E_s}{E_P}}$$

In general-

$$\boxed{\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = K}$$

Here K = transformation ratio

Note -

If $N_s > N_p$ then $E_s > E_p$ then the transformer is called step up transformer.

If $N_s < N_p$ then $E_s < E_p$ then the transformer is called a step down transformer.

Efficiency of transformer

It is defined as the ratio of output power to the input power.

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100\,\%$$

$$\boxed{\eta = \frac{E_s I_s}{E_p I_p} \times 100\,\%}$$

Note -

The transformer cannot work on d.c. A transformer changes a.c. voltages/currents. It does not affect the frequency of a.c. when a.c. voltage is raised n times, the corresponding alternating current reduces to 1/n times.

Energy Losses in a transformer

(i) Copper Loss -

It is the energy loss in the form of heat in the copper coils of a transformer. This is due to Joule heating of conducting wires. These are minimised using thick wires.

(ii) Iron Loss -

It is the energy loss in the form of heat in the iron core of the transformer. This is due to formation of eddy currents in iron core. It is minimised by taking Laminated cores.

(iii) **Leakage of magnetic flux -**

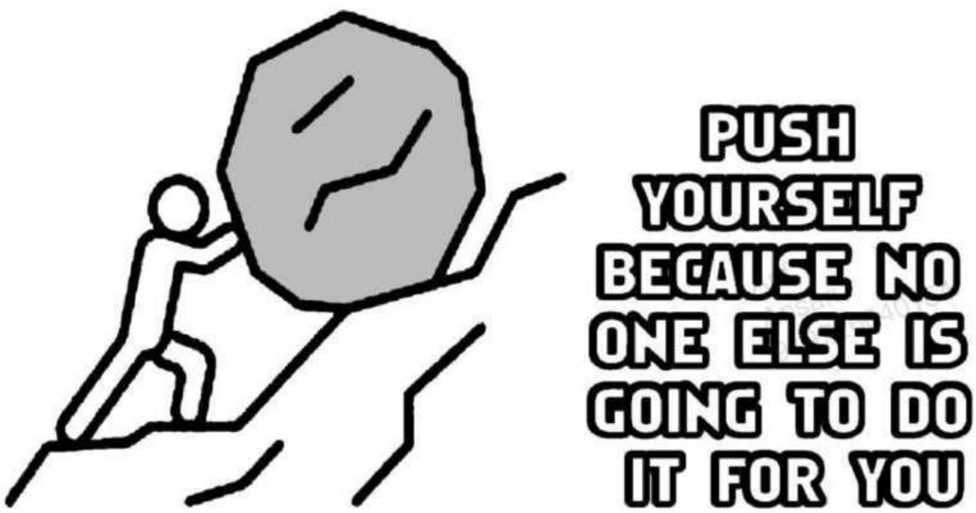
Due to this the rate of change of magnetic flux linked with each turn of S_1S_2 is less than the rate of change of magnetic flux linked with each turn of P_1P_2. It can be reduced by winding the primary and secondary coils one over the other.

(iv) **Hysteresis Loss -**

This is the loss of energy due to repeated magnetisation and demagnetisation of the iron core when a.c. is fed to it. The loss is kept to a minimum by using a magnetic material which has a low hysteresis loss.

(v) **Magnetostriction -**

It is humming noise of a transformer. Therefore, output power in the best transformer may be roughly 90% of the input power.



Q. CBSE 2017

Calculate the current drawn by the primary of a transformer which step-down 200 V to 20 V to operate a device of resistance 20 Ω. Assuming the efficiency of the transformer to be 80 %.

Sol. Efficiency of transformer $= \dfrac{\text{Output Power}}{\text{Input Power}} \times 100$

$$80 = \frac{V_s I_s}{V_p I_p} \times 100$$

$$4 V_p I_p = 5 V_s I_s$$

$$4 V_p I_p = 5 V_s \times \left(\frac{V_s}{R_s}\right) \qquad \left[I_s = \frac{V_s}{R_s} \right]$$

$$4 \times 200 \times I_p = 5 \times 20 \times \frac{20}{20}$$

$$I_p = \frac{1}{8}\ A$$

Q. CBSE 2020

A step-down transformer operated on a 2.5 KV line. It supplies a load with 20A. The ratio of the primary winding to the secondary is 10 : 1. If the transformer is 90 % efficient, then calculate - (a) the power output (b) the voltage (c) the current in the secondary.

Sol. Input Voltage $V_p = 2.5 \times 10^3\ V$

Input Current $I_p = 20\ A$ Also $\dfrac{N_p}{N_s} = \dfrac{10}{1}$

Percentage efficiency $= \dfrac{\text{Output Power}}{\text{Input Power}} \times 100$

$$90 = \frac{\text{Output Power}}{V_p I_p} \times 100$$

(a) Output Power $= \dfrac{90}{100} \times (2.5 \times 10^3 \times 20)$

$= 4.5 \times 10^4\ W$

(b) $\because \dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} \Rightarrow V_s = \dfrac{N_s}{N_p} \times V_p$

$$V_s = \frac{1}{10} \times 2.5 \times 10^3 = 250\ V$$

(c) $V_s I_s = 4.5 \times 10^4\ W$

$\therefore$ Current $I_s = \dfrac{4.5 \times 10^4}{250} = 180\ A$

Q.1. The peak value of an alternating current is 5A and its frequency is 60 Hz. Find its rms value. How long will the current take to reach the peak value starting from zero?

Solution:

The rms current is $i_{rms} = \dfrac{i_0}{\sqrt{2}} = \dfrac{5A}{\sqrt{2}} = 3.5\,A$

The time period is $T = \dfrac{1}{\nu} = \dfrac{1}{60}\,s$

The current takes one fourth of the time period to reach the peak value starting from zero. Thus, the time required is

$$t = \frac{T}{4} = \frac{1}{240}\,s$$

Q.2. An alternating emf is represented by $E = 100 \sin(120\pi t + \pi/4)$ volt. Calculate
(i) Average or mean value of emf
(ii) RMS value of emf
(iii) Frequency of alternating emf
(iv) the shortest time interval after start at which emf is zero.

Solution :

(i) $E_{av} = \dfrac{2E_0}{\pi} = 0.636\ E_0$

$= 0.6365 \times 100 \text{ volt} = 63.65 \text{ volt}$

(ii) $E_{rms} = \dfrac{E_0}{\sqrt{2}} = \dfrac{100}{\sqrt{2}} = 70.7 \text{ volt}$

(iii) On comparing with $E = E_0 \sin(\omega t + \phi)$, we get $\omega = 120\pi, \quad \Rightarrow 2\pi f = 120\pi \quad \Rightarrow f = 60\,Hz$

(iv) Substituting zero for instantaneous value of emf

$$0 = 100 \sin(120\pi t + \pi/4)$$

$$\Rightarrow \sin\left(120\pi t + \frac{\pi}{4}\right) = 0$$

For value of t to be positive, $120\pi t + \frac{\pi}{4} = \pi$

$$\Rightarrow t = \frac{3\pi}{4} \times \frac{1}{120\pi} \text{ sec} = 6.25 \times 10^{-3} \text{ sec.}$$

Q.3. A pure inductor of 50.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Solution:

The inductive reactance.

$$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 50 \times 10^{-3}\,\Omega$$

$$= 15.7\,\Omega$$

The rms current in the circuit is

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220V}{15.7\Omega} = 14.01A.$$

Q.4. An LCR circuit contains resistance of 100 Ω and supply of 200 V at 300 rad/sec. If only capacitance is taken out from the circuit and the rest of the circuit is joined, current lags behind the voltage by 60°. If on the other hand, only inductor is taken out, the current leads by 60° with applied voltage. Find the current flowing in the circuit.

Solution :Resistance* R *= 100 Ω

Applied voltage $V_0 = 200$ volt

When the capacitance is taken out, only inductor and resistor will remain in circuit.

$$\text{The phase } \tan(\phi) = \frac{X_L}{R}$$

$$\tan(60^\circ) = \frac{X_L}{R} \quad X_L = \sqrt{3}R$$

When the inductor is taken out, only capacitor and resistance will remain in circuit.

Then phase $\tan(\phi) = \frac{X_C}{R}$

$\tan(60^\circ) = \frac{X_C}{R}$ $X_C = \sqrt{3}\ R$

Impedance

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = R = 100$$

[since $X_C = X_L = \sqrt{3}\ R$]

Current in the circuit $I_0 = \frac{V_0}{Z} = \frac{200}{100} = 2\ A.$

Q.5. ***An inductance of 2.0 H, a capacitance of 18 μF and a resistance of 10 kΩ are connected to an ac source of 20 V with adjustable frequency.***

(i) At what frequency, will the current be maximum in the circuit ?

(ii) What is this maximum current.

Solution:

(i)Current will be maximum at resonant frequency f_r.

$$f_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2\times18\times10^{-6}}} = 26.5\text{Hz}$$

(ii) At resonance R = Z.

At resonant frequency,

$$I_{max} = \frac{E_{max}}{Z} = \frac{E_{max}}{R} = \frac{20\sqrt{2}}{10000\,\Omega} = 2.83\,\text{mA}$$

Q.6. ***An L-C-R series circuit is connected to an external emf E= 200 sin (100 π t). The values of the capacitance and resistance in the circuit are 1μF and 100 Ω respectively. Find the inductance for which current in the circuit is maximum.***

Solution: Current in circuit

$$I_0 = \frac{V_0}{Z} \quad \text{where } Z = \sqrt{R^2 + (X_C - X_L)^2}$$

For current I_0 to be maximum, impedance Z must be minimum Z will be minimum, when

$$X_C = X_L \qquad \text{i.e. } \frac{1}{\omega C} = \omega L \qquad L = \frac{1}{\omega^2 C}$$

$(\omega = 100\pi$ from $E = 200 \sin(100\pi t))$

$$L = \frac{1}{(100\pi)^2 \times 10^{-6}} = \frac{1}{(100\pi)^2 \times 10^{-6}} = \frac{100}{\pi^2} H = 10H$$

Q.7. A power transformer is used to step up an alternating emf of 200 V to 4 KV and to transmits 5 KW power. If the primary is of 1000 turns, calculate, assuming the transformer to be ideal.

(i) The number of turns in the secondary

(ii) The current rating of the secondary

Solution:

(i) For an ideal transformer $\frac{e_2}{e_1} = \frac{N_2}{N_1}$

$$\frac{4000}{200} = \frac{N_s}{1000} \qquad \Rightarrow \quad N_s = 2 \times 10^4 \text{ turns}$$

(ii) P = (power to be transferred) = $e_1 i_1$

$$\Rightarrow i_1 = \frac{P}{e_1} = \frac{5 \times 10^3}{200} = 25 \text{ A}$$

then $\frac{i_2}{i_1} = \frac{N_1}{N_2} = \frac{10^3}{2 \times 10^4} = \frac{1}{20}$

$$\therefore i_2 = (i_1)\left(\frac{1}{20}\right) = \frac{25}{20} = 1.25\text{A} \quad \text{or}$$

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Alternatively, $e_2 i_2 = 5 \times 10^3$ W

$$\therefore \quad i_2 = \frac{5 \times 10^3}{4 \times 10^3} = 1.25\text{A}$$